12. *Applications to some differential equations* (38 pages): Several examples are given (a.o. Riccati and (almost) Euler-Cauchy) and a very recent method of solving the Riccati equation using *T*-fractions (due to S. C. Cooper) is treated.

MARCEL DE BRUIN

R. A. LORENTZ, *Multivariate Birkhoff Interpolation*, Lecture Notes in Mathematics, Vol. 1516, Springer-Verlag, 1992, ix + 192 pp.

Multivariate polynomial interpolation is a subject which always has attracted much attention, because it is basic in many other mathematical problems: finite elements, splines, cubature formulas, etc. Many papers and theses have been written on this subject in the last decade, but a clear convincing theory, accessible to practitioners, is yet to be achieved. Due to this, most of the textbooks on numerical analysis or approximation theory omit the problem or include only a few pages on it. There is a lack of books entirely devoted to the subject. The present book under review can be considered as a natural continuation of the book *Birkhoff interpolation* by G. G. Lorentz, K. Jetter, and S. Riemenschneider (Addison–Wesley, 1983), which deals with univariate Birkhoff interpolation problems, that is problems with derivatives of any order as interpolation data.

R. A. Lorentz presents his own research on the subject in a good part of the book and some other points of view in the rest, including a long list of references which, if not exhaustive, will be very useful to the reader. The first of the 13 sections is introductory and is followed by 6 others in which the author extends the univariate techniques to multivariate problems. After a section of examples from the theory of finite elements there are two sections (9 and 10) in which the results of the previous sections are applied to Hermite problems, that is problems with the same number of data at every node. In Section 11 several ways of computing Vandermonde-like determinants arising in multivariate interpolation problems are given. The last two sections deal with approaches where the dimension of the interpolation space is greater than the number of interpolation data and some extra conditions can be imposed in order to guarantee a unique solution.

The emphasis is on theoretical rather than practical aspects of the problems, more precisely on discussing if their solvability depends on the choice of the set of nodes and derivatives. Most of the interesting problems are almost regular; i.e., they are solvable for almost all choices of nodes. But it is difficult to identify in advance, in a practical way, the negative choices. Very few pages are devoted to constructive approaches, and the reader should not expect a collection of methods of solution for many problems. But this is a book, written in a very clear style, that every mathematician interested in multivariate interpolation must know.

MARIANO GASCA

E. M. NIKISHIN AND V. N. SOROKIN, *Rational Approximations and Orthogonality*, Translations of Mathematical Monographs, Vol. 92, American Mathematical Society, 1991, viii + 221 pp.

The concept of rational approximation forms the background of this book. In the framework of number theory it leads to classical Diophantine approximations, while in function theory it gives rise to Padé-type approximants. For the latter the notion of orthogonality plays a central role. This approach allows one to connect the theory of Padé approximants via the theory of orthogonal polynomials with different branches of mathematics such as operator theory and mathematical physics. Some additional techniques from the theory of boundary values of analytic functions and potential theory are the advanced tools of this investigation.

Chapter 1 of the book is devoted to rational approximations of numbers with a main goal consisting of a methodologically new proof of Roth's theorem. Chapter 2 deals with Padé approximants and orthogonal polynomials. The authors describe the main links between these notions, including continued fractions and the moment problem. The last sections in this chapter give applications to operator theory and some problems of mathematical physics. Chapter 3 presents asymptotic properties of orthogonal polynomials with a modern proof of the well-known Szegő asymptotic formula. In Chapter 4 the Hermite–Padé approximants are treated, from the classical results of Hermite and multidimensional continued fractions up to the latest achievements in this field. Chapter 5 contains the necessary background of modern potential theory required for the study of the asymptotics of rational and Hermite–Padé approximants. The book is a comprehensive introduction for students and future researchers to topics that are nowadays undergoing intensive development.

A. I. APTEKAREV

CHARLES K. CHUI, *An Introduction to Wavelets*, Wavelet Analysis and Its Applications, Vol. 1, Academic Press, 1992, x + 264 pp.

Wavelet theory emerged only recently as a distinguished piece of abstract and of applied mathematics. For quite a while, Fourier analysis has been an essential tool in signal processing and many other applications for that matter. The Fourier transform (continuous or discrete) decomposes the function (signal) in a range of low to high frequency components, corresponding to the smooth to highly oscillating parts of the function. Fourier analysis has the essential property that functions which have a narrow support in one domain (e.g., a pulse) will have a broad support in the transform domain (e.g., a sine wave). One of the basic ideas of wavelet analysis is to have basis functions with compact support in both domains.

The deficiency of classical Fourier analysis led signal processing engineers to concepts like short term Fourier transforms, filter banks, etc., while the abstract harmonic analyst ventured in the Calderón–Zygmund theory, the theory of frames, and the like. Wavelet theory is where their divergent trails met again in perfect harmony. Multiresolution analysis forms a natural tool, both for the applications and for the theory. To cover the whole space of signals, some "scaling function" with compact support and with a limited frequency content is translated (to cover the whole time axis) and scaled or dilated (to cover all frequencies) and together they form a basis. The scaling factor is a parameter that allows one to zoom in or out on the high frequency components of the signal.

Several books and long and excellent survey papers on the subject are already available at the moment. However, many of the books are proceedings or a collection of papers presenting different angles of approach to the subject. The present volume is one of the first to give a self-contained introduction, not only to the subject itself and the related topics mentioned above, but also to the essential building blocks like Fourier and spline analysis. Splines are not essential for wavelets, but as a most elegant tool to represent locally supported functions, it is not surprising that spline–wavelets are a natural breed. The author has put in this volume the emphasis on this type of wavelets. An important impetus for the breakthrough of wavelet theory was the discovery of the fact that the wavelet idea could be married with the fact that the basis of dilations and translations could be made orthonormal (or some weaker form of this) with the preservation of all the nice properties one had before. This aspect is studied in the last chapter of the book.

The book can safely be recommended to the wavelet illiterate when he has some basic knowledge of real analysis and function theory. The development is strongly biased towards the spline wavelets though. The applications are always right below the surface, but the material in the book is essentially mathematics.

ADHEMAR BULTHEEL